**Investments Answers to Problem Sheet 7 Lent Term 2024**

**True-false**

1. If interest rates rise, bond prices fall.

**True. Price is the present value of cash flows, so if the interest rate at which the cash flows are discounted rises, the value of the bond falls.**

2. The value of a portfolio that has zero duration is unaffected by small interest rate changes.

**True. Provided the shift in interest rates is parallel across the maturities held in the portfolio.**

3. If a bond is trading above par (i.e. its market value exceeds its face value) then its interest or running yield is higher than its yield to maturity.

**True. The return to holding a bond comes from the interest yield and the capital gain. If the bond is above par, the latter component must be negative over the life of the bond, so the average yield received over the life, which is the yield to maturity, is below the interest yield.**

4. The price of a five year bond with a 3% coupon should be exactly half way between the price of two other five year bonds, one with zero coupon, and one with and a 6% coupon.

**True. The cash flow of £200 nominal of the 3% bond is identical to that of a portfolio consisting of £100 nominal each of the other two bonds.**

5. In general, you get a higher rate of return from long-dated bonds than from short-dated bonds.

**Possibly true. This is the liquidity preference theory. This is plausible if you believe that investors tend to have a shorter horizon than borrowers, and hence require a premium to persuade them to hold longer-dated bonds. The empirical evidence supports the idea, but it is quite weak statistically.**

6. If forward rates are higher than spot rates, yields on longer-dated bonds are higher than yields on shorter-dated bonds.

**True. If *r1* is one year spot rate, and *f1,2* is the one year forward rate one year forward, then (1+*r2*)2 = (1+*r1*)(1+*f1,2*)so if *f1,2 > r1* it is clear that *r2 > r1*. In words, a long term spot rate is an average of the short term spot rate and forward rates, so the result follows.**

8. The dirty price of a bond tends to be more variable than the clean price.

**True. The dirty price rises (the bond becomes more valuable) as the coupon date approaches, and then it drops as the bond goes ex-dividend. The clean price is more or less unaffected by these changes over the coupon cycle, and its volatility is due almost entirely to changes in the general level of interest rates.**

**Problems**

1. The table below lists the prices of zero-coupon bonds of various maturities, expressed as a price per $1000 of principal. Calculate the yields to maturity of each bond and the sequence of implied forward rates.

|  |  |
| --- | --- |
| Maturity (years) | Price of bond ($) |
| 1 | 943.40 |
| 2 | 898.47 |
| 3 | 847.62 |
| 4 | 792.16 |

Assuming that the *pure expectations hypothesis* is valid, calculate the expected price path of the four-year bond as time passes. What is the rate of return of the bond in each of the four years?

**The yields and forward rates use formulae from the lecture. For example the yield on the four year bond is given by:**

**1000/(1+*y*)4 = 792.16**

**giving *y* = 6.00%. The forward rates are computed as follows: $1000 in year 3 is worth $847.62 now. $1000 in year 4 is worth $792.16. So the implied interest rate from year 3 to year 4 is 847.62/792.16-1 = 7%.**

**If the expectations hypothesis holds, the forward rate is the expected future spot rate, so we can forecast the future price of the bond by discounting its pay-off by the relevant forward rates. The expected return on the bond is then the forward rate (eg the expected return on the bond in the third year is 6.0%).**

**The bond will be worth $1000 at the end of year 4. If the one year rate at the beginning of year 4 is equal to the forward rate of 7%, the bond will be worth $1000/1.07 = $934.57. If the one year yield at the beginning of year 3 is the forward rate of 6%, the bond price will be the end year price discounted at 6%; this is $934.57/1.06 or $881.68. And so on.**

2. The yield-to-maturity on one-year and two-year zero-coupon bonds is currently 7% and 8%, respectively. The Treasury plans to issue a two-year bond that pays an annual coupon of 9% of face value. The face value of the bond is £100.

a. At what price will the bond sell?

**The bond is equivalent to £109 of 2-year zero coupon bond and £9 of 1-year. It is worth 9/1.07 + 109/1.082 = £101.86.**

b. What will be the yield-to-maturity of the bond?

**The yield solves 9/(1+y) + 109/(1+y)2 = 101.86, which gives a yield of 7.96%.**

c. If the *pure expectations hypothesis* for the term structure of interest rates is correct, what is the market expectation of the price that the bond will sell for next year?

**The one year spot rate next year is expected to be today’s forward rate of 93.46/85.73-1, or 9.01%. So the price of the bond is expected to be £109/1.0901 or £99.99.**

d. Repeat the calculation in (c) assuming the *liquidity preference hypothesis* holds with a liquidity premium of 1%.

**If there is a 1% premium in the longer rate, then the expected future spot is 8.01%, and the expected bond value is £100.92.**

**The calculations are also done in the following embedded spreadsheet:**

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3.Without doing any detailed computations, and assuming that the term structure is flat at 5%, put the following bonds in order of increasing duration:

(a) a perpetual 5% coupon bond

(b) a perpetual 3% coupon bond

(c) a 25 year zero coupon bond

(d) a 10 year floating rate bond that pays LIBOR

(e) a 5 year 5% coupon bond

(f) a 5 year 6% coupon bond

**A perpetual bond with yield *y* has a price of *P =* c/*y* so *Dmod = -*(1/*P*) *dP/dy* = 1/*y.* So (a) and (b) will both have durations of about 20 years. (c) will have a duration of 25 years since all its cash flows are at that time, and a slightly smaller modified duration (25/1.05 = 23.8 years). (d) will have a duration equal to the time to the next coupon, so no more than a year. (e) and (f) will both have durations of somewhat under 5 years, with (f) having more of its value being paid in coupons before maturity and so have a shorter duration. The order is thus d, f, e, a=b, and then c.**

4. The YTM on 1-year zero-coupon bonds is 5% and the YTM on 2-year zero-coupon is 6%. The YTM on 2-year coupon bonds with coupon rates of 12% (paid annually) is 5.8%. What arbitrage opportunity is available for an investment banking firm? What is the profit on the activity?

**Answer:**

The price of the coupon bond, based on its yield to maturity, is:

[$120 × Annuity factor (5.8%, 2)] + [$1,000 × PV factor (5.8%, 2)] = $1,113.99

If the coupons were stripped and sold separately as zeros, then, based on the yield to maturity of zeros with maturities of one and two years, respectively, the coupon payments could be sold separately for:



The arbitrage strategy is to buy zeros with face values of $120 and $1,120, and respective maturities of one year and two years, and simultaneously sell the coupon bond. The profit equals $2.91 on each bond.